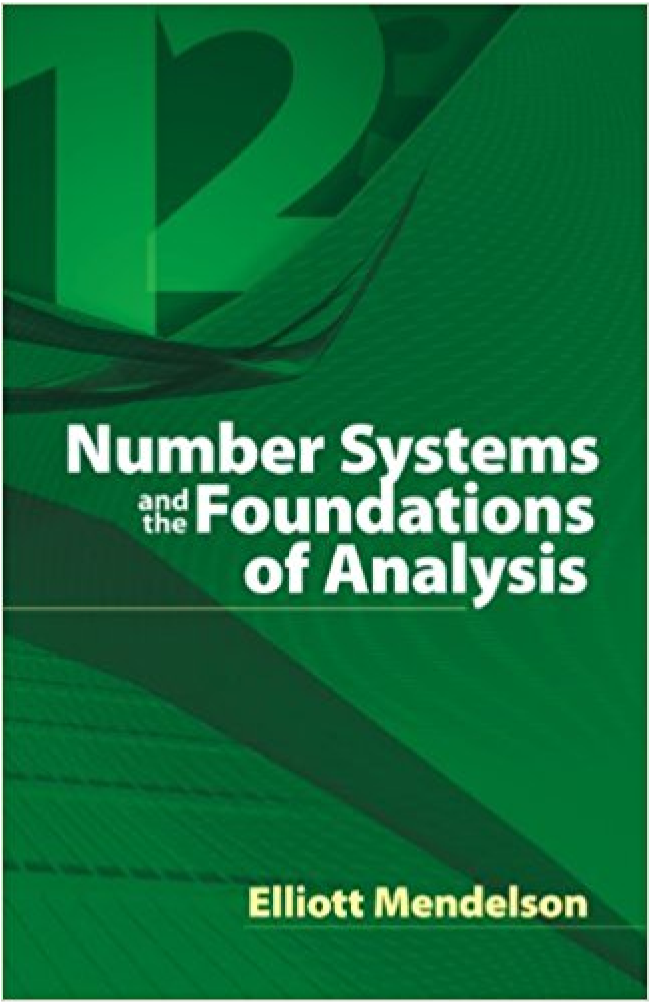
Number Systems And The Foundations Of Analysis (Dover Books On Mathematics)



Synopsis

This study of basic number systems explores natural numbers, integers, rational numbers, real numbers, and complex numbers. Written by a noted expert on logic and set theory, it assumes no background in abstract mathematical thought. Undergraduates and beginning graduate students will find this treatment an ideal introduction to number systems, particularly in terms of its detailed proofs.Starting with the basic facts and notions of logic and set theory, the text offers an axiomatic presentation of the simplest structure, the system of natural numbers. It proceeds, by set-theoretic methods, to an examination of integers that covers rings and integral domains, ordered integral domains, and natural numbers and integers of an integral domain. A look at rational numbers and ordered fields follows, along with a survey of the real number system that includes considerations of least upper bounds and greatest lower bounds, convergent and Cauchy sequences, and elementary topology. Numerous exercises and several helpful appendixes supplement the text.

Book Information

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Customer Reviews

IntroductionI am currently reading this book to refresh my high school algebra and so far this book is doing a pretty good job. This book should be the assigned textbook for all honors high school students taking either pre-algebra or algebra 1. This book is currently being assign for Math 317/617 at queens college. According to the preface, no mathematical training is presupposed. In fact, the reader does not need to know that 2 + 2 = 4 since it will be proved. However, the reader needs to be familiar with formal derivations involving quantifiers. For example, at the level of Chapters 5 and 10 fromÂ The Logic Book (4th Edition). Since Mendelson invokes Zermelo-Fraenkel set theory, I highly recommend the reader to supplement this textbook withElements of Set Theory. Enderton's book is also pretty Damm good. Summary of Chapters 1, 2, B, E, DChapter 1 covers basic notions of logic and naÃƒÂ ve set theory. However, Appendix E can be supplemented to follow the Zermelo-Fraenkel axiomatic version for sets (which is what i did). The following axioms are introduced: (1)Extensionality, (2)Empty set, (3)Pairing, (4)Union, (5)Restricted comprehension, (6) power set and (7) Infinity. Equipped with these axioms, Mendelson shows how to derive the existence of a Peano system. The set P is a Peano system if and only if there exists sets N, S, 1 such that P = with the following properties. (A) S is a one to one function from N into N. (B) 1 is contained in N. (C) 1 is not contained in the range of S. (D) If A is a subset of N with 1 contained in A such that whenever an arbitrary set x is in A implies that S(x) is in A, then A=N.The recursive/iteration theorem is then verified and followed by the proof that any two Peano systems are isomorphic. I did not like the proof that Mendelson gives for the recursive/iteration theorem and so I suggest the reader to see Enderton's proof instead (see Enderton pg. 73). I find Enderton's proof of the recursive/iteration theorem more natural and straightforward than Mendelson's proof. Mendelson also gives a second form of the recursive/iteration theorem which he calls the recursion theorem. The proof of the recursion theorem that Mendelson gives is straightforward and vital to construct interesting functions like the factorial and sigma functions. Basic arithmetic like Addition, Multiplication and Exponentiation is also covered. Appendix D covers the basics of finite and infinite sets as well as finite and infinite sequences. Mendelson proves the Schroder - Bernstein theorem for the case where the sets are disjoint but leaves it to reader for case where the sets are not disjoint (which sucks). Again, see pg. 147 in Enderton's book for a complete detail proof. Section B on sigma sums is very short. For more theorems involving sigma sums (like the binomial theorem), see section 4.9 fromÂ The Number Systems of AnalysisÂ or sections 3.4 and 4.4 fromÂ The Number Systems: Foundations of Algebra and Analysis. Note that whatever is proofed in the 3 mentioned books also applies to this book since all of these books are using the same axiomatic system and because they have the same definitions for the most part. Summary of chapter 3Chapter 3 is about constructing a well-ordered integral domain. Mendelson offers two constructions. One of them is using equivalence class sets. The other one is using a more direct approach. (You have to read the book to find more about it). Regardless of which approach the reader takes, Mendelson shows that any two well-order integral domains are isomorphic. Chapter 3 also covers basic properties of the integers such as greatest common divisors, arithmetic, and primes. Chapter 3 however lacks properties of sigma notation and polynomials. Again see the books previously mentioned for additional material. There is one crucial thing that i didn’t like about chapter 3. Mendelson does not proves the following theorem. If R is an equivalence relation on B and F: B X B -> B such that xRy and mRn implies F()RF(), then there exists a unique function H: B/R X B/R -> B/R such that H(< [a] , [b] >) = [F(< a , b >)] where a and b are elements of B. This theorem is crucial for justifying the uniqueness and existence of the functions +z , *z , +Q , *Q. See Enderton pg. 61 and 62 for further discussion. Summary of chapter 4 and 5I am currently reading these chapters and so far the rational numbers are constructed using equivalence class sets. The real numbers are constructed using two alternate methods. One method is the Cauchy method. The other method is by using the so called "Dedekind Cuts". Regardless of which method is employed, the author shows that any two complete order fields are isomorphic. Furthermore, Chapter 5 starts looking like any other standard text on Real Analysis. Contrary to most standard texts on real analysis that tackles Real Analysis out of thin air, this book tackles Real Analysis from Zermelo’s axiomatic system. ConclusionI highly recommend this book to anyone interested in learning how to construct the number systems using some of the ZF axioms. And also for those who wish to refresh their high school algebra. If you want to become a God-Damm math genius for less than $20, then this is the book to buy.

This book is very comprehensive. It contains both the Cauchy sequence method and the Dedekind cut method of developing the real number system. (It even points you to books giving the decimal expansion development!) The description of power series gives a rigorous development of decimal expansions. It’s treatment of integral domains shows how facts about natural numbers, integers and rational numbers carry over to the reals. And the treatment of the Archimedean property is excellent. It actually treats the foundations of analysis by proving the basic facts about open and compact sets and continuous functions. It’s the perfect book on number systems for the undergraduate since it starts with basic logic.

A good text book. Set up more for a two semester course on the construction of number systems moving into a standard beginning analysis course. The writing and the organization of the text are excellent. It’s not as terse as Landau’s text. This one has more details on sets and proofs than Thurston’s text (and more examples and exercises, too). Depending on your logical skills, the book could ostensibly be used for self-study, but it was designed to be used in a course with an experienced instructor.If you are a beginning student of advanced mathematics, I suggest reading

Thurston's book, the first half first. Then either begin working in Mendelson's book or read Thurston's book in the chapter order he recommends in his preface (that includes re-reading the first half). Theoretically you can read use each book in tandem to help expand upon your understanding (Mendelson has better examples and better explanation of the formalisms).